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Nelson–Brown Motion: Some Question Marks

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It is shown that no consistent classical interpretation of quantum mechanics was given by Edward Nelson in his paper of 1966: "Derivation of the Schrödinger equation from Newtonian mechanics."

1. INTRODUCTION

From time to time the ideas of quantum mechanics are challenged by attempts at reinstalling the classical interpretation. One of the best known of those attempts was presented by Nelson (1966). As found in that paper there is a close formal analogy between the time evolution of a Schrödinger wave packet and a classical stochastic process. The analogy is found by considering a hypothetical classical point particle whose position $\bar{x}(t)$ is a random variable satisfying the following stochastic differential equations:

$$d\bar{x}(t)=\left[\bar{v}(\bar{x},t)+\bar{u}(\bar{x},t)\right]dt+d\bar{w} \qquad (dt>0)$$
$$d\bar{x}(t)=\left[\bar{v}(\bar{x},t)-\bar{u}(\bar{x},t)\right]dt+d\bar{w}^{*} \qquad (dt<0) \tag{1.1}$$

The motion is assumed to take place in a classical external potential $V(\bar{x},t)$. The quantities $\bar{v}(\bar{x},t)$ and $\bar{u}(\bar{x},t)$ are the fields of drift and osmotic velocities and $d\bar{w}$ and $d\bar{w}^{*}$ are random position shifts due to certain Wiener processes. Denoting the probability density for $\bar{x}(t)$ by $\rho(\bar{x},t)$ and assuming that the stochastic motion is "in equilibrium" one shows that $\bar{u}(\bar{x},t)=$

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$(\hbar/2m)\overline{\nabla}[\ln \rho(\bar{x},t)]$. Postulating, as a dynamical principle, Newton's second law for the mean values of force and acceleration, and moreover, assuming that $\bar{v}(\bar{x},t)$ has a potential $(\hbar/m)S(\bar{x},t)$ one proves (Nelson, 1966) that the complex function $\Psi(\bar{x},t)=\rho(\bar{x},t)^{1/2}\exp[iS(\bar{x},t)]$ fulfills the Schrödinger wave equation with the external potential $V(\bar{x},t)$. As suggested by Nelson, this means that the "wave function" of a Schrödinger particle is just a complex representation for the real quantities $\bar{u}(\bar{x},t)$ and $\bar{v}(\bar{x},t)$ characterizing a classical random motion. Nelson concludes that the transition to quantum mechanics around 50 years ago was unnecessary. The true physical object which appears in the known quantum mechanical experiments might as well be interpreted as a classical point particle undergoing a generalized random motion. The formal aspects of Nelson's paper are so attractive that they, somehow, overshadow its physical content. After looking at that content more closely, however, we have found the physical ideas in Nelson (1966) so elusive that the very statement about the classical reinterpretation must be questioned.

2. POINT OR FIELD?

The first question to ask is the fundamental one. What is the physical object in Nelson's theory? According to Nelson himself his formalism describes simply a classical point particle undergoing a generalized Brownian motion with no friction. A statement at the end of Section III of his paper makes clear, however, that the "point particle" possesses some fieldlike degrees of freedom. One can read (Nelson, 1966, p. 1082):

> The state of the particle at time t_0 is described by its position $\bar{x}(t_0)$ at time t_0, the osmotic velocity $\bar{u}$ at time t_0, and the current velocity $\bar{v}$ at time t_0. Notice that $\bar{u}(\bar{x},t_0)$ and $\bar{v}(\bar{x},t_0)$ must be given for all values of $\bar{x}$ and not just for $\bar{x}(t_0)$.

As a consequence, in the Nelson equations of Brownian motion there appears a complex function $\Psi(\bar{x},t)$ [simply related to $\bar{u}(\bar{x},t)$ and $\bar{v}(\bar{x},t)$]. A question thus arises, of what this function physically means. What are the "drift and osmotic velocities" $\bar{u}(\bar{x},t)$ and $\bar{v}(\bar{x},t)$? Are they of external origin [as in genuine Ornstein–Uhlenbeck theory (Chandrasekhar et al., 1954)] or are they, somehow, created by the random motion itself? Depending on the answer to this question, two physical interpretations of Nelson's scheme might be attempted.

2.1. Trajectory Interpretation. The only physical system behind the Nelson formalism is just a classical point particle. The quantities ρ, $\bar{u}$, $\bar{v}$, and S have no physical reality of their own. They are just "painted" by the random trajectories. The stochastic process develops by constructing its own $\bar{v}(\bar{x},t)$ and $\bar{u}(\bar{x},t)$ [or equivalently, $\Psi(\bar{x},t)$] which, in turn, serve as an initial condition for the further development of the process.

2.2 Field Interpretation. The quantities $\bar{u}(\bar{x},t)$ and $\bar{v}(\bar{x},t)$ are not created by the random motion, but they represent an external reality in which the motion takes place (as in the original Ornstein–Uhlenbeck theory of Brownian motion). Consistently, $\Psi(\bar{x},t)$ is a physical field intervening between the external potential and the point particle. This field is conditioned by $V(\bar{x},t)$ (via the Schrödinger equation) and it conditions, in turn, the Brownian trajectory by creating the fields of osmotic and drift velocities which the random motion must obey.

3. FAILURE OF THE TRAJECTORY INTERPRETATION

If the first interpretation is adopted, one must still ask whether the quantities $\bar{u}$ and $\bar{v}$ (and therefore Ψ) are created by each single "point particle" or if they are defined only for a wider ensemble? Following traditions of a statistical theory one would rather like to interpret Ψ in terms of an ensemble [so that $\rho(\bar{x},t)$ would mean an average density of the ensemble particles and $\bar{u}(\bar{x},t)$, $\bar{v}(\bar{x},t)$ and $S(\bar{x},t)$ would be related to the particle currents]. Then, however, the appearance of $\bar{u}(\bar{x},t)$ and $\bar{v}(\bar{x},t)$ [and consistently, of $\rho(\bar{x},t)$ and $S(\bar{x},t)$] in the equations of motion (1.1) for each single trajectory would mean that one does not have a true statistical ensemble (i.e., an ensemble composed of independent particles) but rather a cloud of mutually interacting "mass points" so that the form of each Brownian trajectory is affected by the whole rest of the Brownian trajectories. Such phenomena are indeed observed in nonlinear diffusion processes where many diffusing particles interact by modifying the medium in which they propagate [for diffusing clouds dense enough this can even lead to the creation of interference patterns (Turski, 1975)]. However, the problem with the quantum mechanical wave function is that, somehow, it must be valid for each single propagation act, even if it is well separated from the other propagation acts. This has been convincingly shown by the Fabrikant experiment (Biberman et al., 1949). [The chance that there was still some particle clustering overlooked in that experiment is negligible. See also Pfleegor and Mandel (1967) and Faget and Fert (1957) and also the footnote on p. 130 in Jammer (1974)]. Hence, the effects observed in quantum physics cannot be explained by assuming an interaction between many trajectories of different "mass points." A question now arises, what is the sense of drift and osmotic velocities if there is only one trajectory? Is it, perhaps, so that in Nelson's scheme each observed motion of the point particle (real trajectory) is affected by some collection of its own virtual alternatives (ghost trajectories)? However, the idea of a physical process interacting with its virtual alternatives would no longer be classical.[1] One might think that $\bar{u}$, $\bar{v}$, ρ, and S concern a single trajectory but have an expectation meaning [so that, for instance, $\rho(\bar{x},t)$ is the probability density for the position of the Brownian particle). Then, however, it would be strange that the probability density $\rho(\bar{x},t_0)$ enters into the description of the state of a "mass point" together with its coordinates $\bar{x}(t_0)$. This would suggest that the probability distribution $\rho(\bar{x},t_0)$ remains essential even though one already knows the position $\bar{x}(t_0)$ of the particle. One might reply that $\rho(\bar{x},t_0)$ and $\bar{x}(t_0)$ represent just two different levels of information: while $\bar{x}(t_0)$ is precise information possessed by a well-informed observed, $\rho(\bar{x},t_0)$ is merely average information possessed by a less oriented observer. Then, however, it is difficult to understand, how the ignorance of the second observer can affect the further development of the Brownian motion in the eyes of the first observer. And it does in Nelson (1966), because $\Psi(\bar{x},t_0)$ influences the form of the Brownian trajectories which emerge from $\bar{x}(t_0)$. It becomes obvious that in the frame of interpretation (A) there is only one possibility left: that the motion of the point particle is a process with memory and is conditioned by its past. In this case $\bar{x}(t_0)$ could mean the actual position of the point, whereas $\Psi(\bar{x},t_0)$ could represent an "averaged memory" of its past drifting. This, of course, would signify that we do not have here a true Markov process and it would lead to a nontrivial problem of the reinterpretation of the whole Nelson scheme. However, one can show that even such a generalized interpretation (A) would not be sufficient. A counterargument emerges from the paper of Albeverio and Høegh-Krohn (1974). Because of its further implications this argument will be discussed in detail.

Let us start by pointing out that Nelson's scheme works exclusively with the "equilibrium states" for which the probability density for the random variable $\bar{x}(t)$ is a priori assumed to conform to a certain equilibrium distribution: $\Psi(\bar{x},t)=|\rho(\bar{x},t)|^2$. The subsequent arguments about the equivalence of Nelson's theory to the orthodox quantum mechanics are valid only for such states. (The properties of the nonequilibrium states in Nelson's theory are left somewhat obscure.) It is an essential problem, however, whether each single Brownian trajectory indeed tends to create an "equilibrium state." It is also a problem, whether every Schrödinger state can be created in that way. As follows from Albeverio and Høegh-Krohn (1974) this is not always so. A typical

[1]It would also be outside interpretation (A). The hypothetical "ghost trajectories," however unobservable, would nevertheless form a sort of "field" intervening in between the real trajectory $\bar{x}(t)$ and the external potential $V(\bar{x},t)$.

difficulty arises if the wave function Ψ vanishes on some surfaces. The equations (1.1) for the random trajectory do not then permit the "Brownian particle" to penetrate, with nonvanishing probability, the nodal surface $\Psi = 0$. This creates an inconsistency between the behavior of the single trajectory and the assumed equilibrium state. As an example take $\Psi = \Psi_0$ to be a spherically symmetric wave function, which is stationary and vanishing on a sphere $r = r_0$ (for example, Ψ_0 may be one of the eigenstates of the hydrogen atom). Then $\Psi_0 = \Psi_1 + \Psi_2$, where Ψ_1 and Ψ_2 are the two components of Ψ_0 inside and outside the sphere $r = r_0$ (cf. Figure 1). Now, consider a hypothetical Brown–Nelson motion which is supposed to reproduce the quantum mechanical pure state Ψ_0. If the motion started inside (outside) the sphere $r = r_0$, it continues inside (outside) with probability 1. Hence, the random motion can either create only the component Ψ_1 (but then Ψ_2 remains empty) or only Ψ_2 (but then Ψ_1 is nonexistent). Consistently, every statistical ensemble of Brown–Nelson trajectories should split into two subensembles: those that reproduce Ψ_1 and those that reproduce Ψ_2. This means that not even a statistical ensemble of "Brownian particles" should be able to imitate the pure state Ψ_0. This should be detectable by arranging any experiment in which the interference between the components Ψ_1 and Ψ_2 would intervene: the inability of the Brown–Nelson trajectories to "construct" the whole of $\Psi_0 = \Psi_1 + \Psi_2$ should then be felt as the lack of the interference effects. Thus, the trajectory interpretation of Ψ_0 bears a certain intrinsic contradiction: starting from the assumption that an ensemble of Brownian trajectories "creates" the pure state Ψ_0 one ends up with the conclusion that the state, in fact, is mixed.

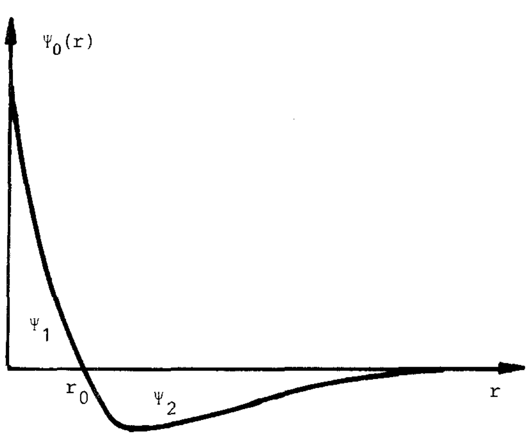

Fig. 1.

The difficulty of nodal surfaces has been noticed by Nelson but it was underestimated by him. Nelson (1966) suggests that the undesired effects occur just as an exception, for Ψ_0 having a nodal surface but they disappear if Ψ_0 is slightly perturbed to become a modified wave function Ψ_ε ($\varepsilon \approx 0$); Ψ_ε having no nodal surfaces for $\varepsilon \neq 0$, $\Psi_\varepsilon \to \Psi_0$ for $\varepsilon \to 0$. However, the right conclusion from this argument might be just the opposite. For $\Psi = \Psi_0$ there are physical differences between the hypothetical ensemble of Brown–Nelson trajectories and the corresponding ensemble of Schrödinger particles. By continuity arguments they cannot suddenly arise for $\varepsilon = 0$. Hence, the difficulty must also be present for Ψ_ε ($\varepsilon \neq 0$), even though Ψ_ε has no nodal surfaces. Indeed, one might guess what happens for ε close to zero and Ψ_ε "almost vanishing" at $r = r_0$. The average time needed for the Brown–Nelson particle to pass accidentally from the domain $r > r_0$ to $r < r_0$ (or conversely) then becomes very large. For $\varepsilon \to 0$ this time tends to infinity. Assuming now that the wave function reflects a finite interval of the past history of the particle, one sees that for ε small enough the wave Ψ_ε cannot be "constructed" out of the particle memories. Indeed, if only the average time needed for an accidental transition between two parts of Ψ_ε (relaxation time) exceeds the length of the particle memory, there must occur a "spontaneous reduction" of the wave packet: for various random trajectories, various parts of Ψ_ε will be "dying out" because of not being visited frequently enough. Hence, the assumption that the random trajectory "imitates" a wave packet Ψ_ε can be self-contradictory even if Ψ_ε has no nodal surfaces.[2]

The difficulty above becomes even more essential if one takes into account that the nodal (or "almost nodal") surfaces of Ψ may be due not to an exceptional form of a particular solution (easily perturbed) but to some external conditions (material obstacles, potential barriers). As an example consider an electron wave Ψ propagating inside of a tube. Assume that the tube has a partition inside (cf. Figure 2) which splits the wave Ψ into two separately traveling components Ψ_1 and Ψ_2. Near the end of the tube the partition ends up and Ψ_1 and Ψ_2 join again. Now, if the partition is long enough and the expected time which the material point must spend on one of the sides is longer than the memory, one of the components Ψ_1 and Ψ_2 must vanish and, contrary to the predictions of quantum mechanics, there should be no interference between Ψ_1 and Ψ_2 at the end of the tube in Figure 2. It is thus seen, that even the introduction of a (finite) memory does not allow the trajectory interpretation to describe

[2]It may be, however, that the failure to reproduce the orthodox quantum mechanics at this point is an advantage of Nelson's scheme (in its trajectory interpretation). It cannot be excluded that we are near here to a theory which would be able to describe simultaneously the wavelike propagation and the reduction of the wave packet (the measurement process).

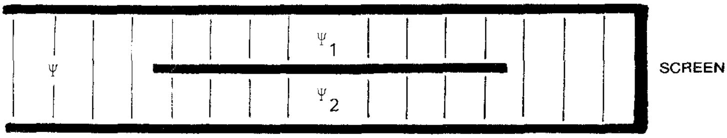

Fig. 2.

properly the orthodox quantum mechanical interference effects.[3] Though it is already out of the original Nelson theory it is worthwhile to notice that the difficulty above would deepen in any attempt to extend the pure point-particle interpretation to describe relativistic quanta like Dirac electrons or Maxwell photons. In fact, in a relativistic theory one can have a wave packet Ψ composed of two space-separated parts Ψ_1 and Ψ_2 ($\Psi = \Psi_1 + \Psi_2$) which remains space separated for some time, even without any external potential (Figure 3).

Suppose now, that one tries to find out a relativistic analog of the Brown–Nelson motion (cf. Lehr and Park, 1977) which would imitate the behavior of such a packet Ψ. Then, one immediately runs into a difficulty of the Albeverio and Høegh-Krohn type. Except if one assumes the possibility of discontinuous trajectories (which already falls outside of the Brown–Nelson type of theory) the particle cannot oscillate in between

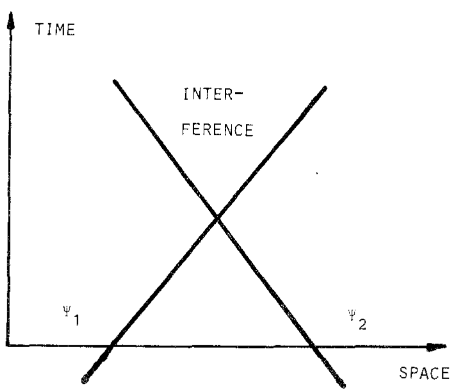

Fig. 3.

[3]There remains a possibility of an infinite memory, which would, however, involve mathematical difficulties. It seems, moreover, that for physically reasonable equations of motion with memory there must be some memory cutoff ascertaining that the far past has a vanishing influence on the actual particle motion.

the two components of Ψ (since with probability 1 it cannot cross surfaces where $\Psi=0$). The motion which started in one of the components must therefore continue (with probability 1) in the same component: hence, the particle trajectory can either "paint" only $\Psi_1(\bar{x},t)$ in the space-time [leaving $\Psi_2(\bar{x},t)$ nonexistent] or only $\Psi_2(\bar{x},t)$ [leaving $\Psi_1(\bar{x},t)$ absent]. Consistently, there should be no interference between $\Psi_1(\bar{x},t)$ and $\Psi_2(\bar{x},t)$ at any later time. An example of this situation is obtained by considering a linearly polarized light beam (or light pulse) $\swarrow\!\!\!\nearrow$ which is decomposed by a crystal of tourmaline into two subbeams $\updownarrow$ and $\leftrightarrow$ corresponding to two perpendicular polarization planes (cf. Figure 4a). Then except if one assumes a radical hypothesis about a material point particle performing some tachyonic jumps between the two propagation branches, there is no possibility that the single random trajectory might construct in space-time the two beams $\updownarrow$ and $\leftrightarrow$. Hence, if one insists on the trajectory interpretation (A) one again ends up with the conclusion of a "spontaneous reduction" of the wave packet: each single act of propagation must choose one of the propagation branches leaving the other branch empty. Consistently, there should be no further interference between the outgoing polarized beams in Figure 4b. However, the experiments show that, in agreement with orthodox quantum theory, *there is* an interference. The polarized light cannot only be decomposed but it can also be unified again, and it then reproduces the original pure state of skew polarization $\swarrow\!\!\!\nearrow$ instead of forming the polarization mixture. One might still object, that the perfect space separation of the two components $\updownarrow$ and $\leftrightarrow$ cannot be achieved in practice, and therefore, the danger of spontaneous reduction does not occur in reality. However, it is not so. In the relativistic wave dynamics the existence of sharply limited wave packets, without asymptotically vanishing tails, is a natural phenomenon and the assumption about the completely space-separated photon beams, in spite of all the diffraction effects

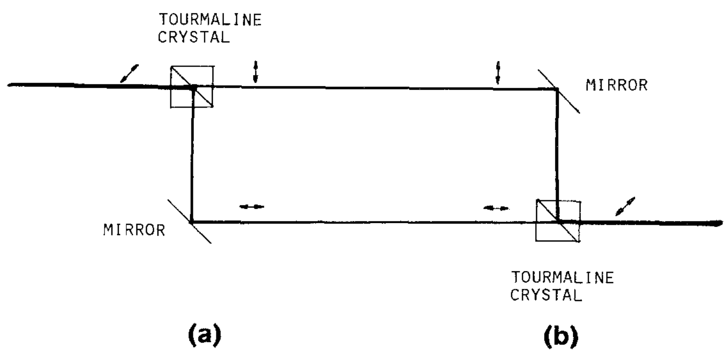

Fig. 4.

is a correct type of idealization. One might again try to save the trajectory interpretation by arguing that each single "classical point particle," although it chooses only one of the propagation branches, conserves nevertheless a "memory" of the branching point, and this memory helps it to become a "skew polarized photon" at the end of the experiment in Figure 4. However, this cannot be an explanation. The final result of the interference experiment in Figure 4 is not only determined by the properties of the branching point. It essentially depends on the whole rest of both propagation branches. By cutting or modifying one of them (e.g., by removing one of the mirrors in Figure 4) one immediately affects the final phenomenon. Hence, not even the existence of a long-term memory can explain the interference experiment. To describe the experiment in terms of a classical point we would have to introduce a nonlocal theory with a hypothetical trajectory which would be "aware" not only of its own past but also of its lost alternatives. Hence, by insisting up to the very end on the trajectory interpretation we would be left with a highly abstract interpretation of Nelson's scheme based on virtual trajectories, as far from any classical model as the present day quantum mechanics. The difficulty which we meet here is not exclusive for Nelson's theory, but is common for all hidden-parameter schemes operating with classical trajectories. *The motion of a classical point-particle conditioned locally by the external potential cannot imitate the quantum mechanical interference effects, no matter whether this motion is strictly deterministic or stochastic with or without memory*. This simple difficulty is deeper and more persistent than all known versions of the theorem of von Neumann (1955) about the nonexistence of hidden parameters. We can only wonder why so many authors fight so hard with the "paper tiger" of von Neumann's theorem that they finally forget about the central difficulty: the one that lies in the interference phenomenon. We conclude that interpretation (A) cannot hold.

4. INCOMPLETENESS OF THE FIELD INTERPRETATION

There is now little choice: we have to examine interpretation (B). Maybe the wave Ψ is not the stochastic motion itself? Perhaps it has an independent existence as a physical field and may not vanish even in the space domains which are never crossed by the random trajectory? This would mean that the true physical object behind the Nelson formalism is not precisely a point particle but a more involved entity: a "stochastic point" plus the "piloting field" Ψ. This, of course, would justify the appearance of both $\bar{x}(t_0)$ and $\Psi(\bar{x}, t_0)$ in the description of the system state. Moreover, this would allow an explanation of the interference effects. Indeed, in the frame of interpretation (B) it brings no difficulty to assume that Ψ is composed of two space-separated parts and that the point particle propagates exclusively in one of them. The other part of Ψ, though devoid of the "material point," does nevertheless exist and whenever unified with the rest of Ψ it can again influence the further propagation of the point. (This is, incidentally, the main advantages of the "pilot wave" variant of the hidden-parameter schemes.) It thus looks as if we here arrive at the ultimate physical meaning of the theorem of Albeverio and Høegh-Krohn (1974): this theorem precisely signifies that the Nelson formalism can describe the interference phenomena only at the cost of considering $\Psi(\bar{x}, t)$ to be a physical field. Formally, this makes Nelson's theory a member of a wider family of hidden-parameter schemes operating with superpotentials and a close relative of a former idea of Bohm (1952a, b). In fact, the difference turns out to be technical rather than fundamental: because Nelson (1966) can now be interpreted as a stochastic variant of Bohm (1952a, b). The price of the reinterpretation, however, is that now the scheme starts to exhibit some physical incompleteness which might also be found in other specimens of the superpotential theory.

First of all, it now becomes even more disquieting that the physical effects in Nelson's theory are discussed exclusively for the "equilibrium states," where the information about the position of the material point inside the wave Ψ is lost and the probability distribution for the random variable $\bar{x}(t)$ becomes $\rho = |\Psi|^2$. If $\Psi(\bar{x}, t)$ and $\bar{x}(t)$ are independent degrees of freedom, the nonequilibrium states are also essential, i.e., states where the information about the classical point was either not completely lost or was partly recovered. In fact, this is precisely the starting point of Nelson himself, who introduces a microstate with a general $\Psi(\bar{x}, t_0)$, but at the same time with a sharply defined $\bar{x}(t_0)$ (corresponding to a δ-like distribution ρ). The next natural step would be to consider a general nonequilibrium state where the probability distribution $\rho(\bar{x}, t)$ is no longer δ-like but has not yet become identical with $|\Psi(\bar{x}, t)|^2$. The statistics of such states would, of course, lead out of the orthodox quantum mechanics. We thus see that even in the frame of interpretation (B) the assertion about the equivalence of the Nelson scheme to the orthodox quantum mechanics has a limited validity: it means that we decide to neglect a priori the nonequilibrium phenomena and the relaxation times. Similar problems have been discussed by, e.g., Bohm (1952a, b, 1953) and Bohm and Vigier (1954).

Interpretation (B), however, shows an even more fundamental incompleteness. If one assumes that Ψ is a physical field, then one must notice that the interaction between the "classical point" and the field Ψ is strictly one-sided. The random trajectory $\bar{x}(t)$ is governed by the field Ψ (via the fields of drift and osmotic velocities). However, the field Ψ does not depend on the random trajectory: it is conditioned exclusively by the external potential $V(\bar{x},t)$. When the wave Ψ divides into several parts, all these parts propagate according to the same law (the Schrödinger equation) no matter which one of them contains the stochastic point. This suggests that we can as well remove the "point particle" completely. The pilot wave Ψ (as it possesses an independent physical reality) will nevertheless exist and will further propagate according to the same Schrödinger equation. This is perhaps one of the most disquieting conclusions from the theories of the Bohm type. Indeed, there is nothing in these theories that prevents the system of pilot wave and point particle from being split so that we could have the pilot wave without the particle inside and the point particle without the piloting wave. This would, however, be a too simple solution of the old dilemma of wave–particle duality. Do we indeed have to believe in the existence of a wavelike ghost of a microsystem devoid of the microsystem itself? Conversely, what happens if we capture the particle and let the wave Ψ "evaporate" to infinity? Can we obtain a "pure" corpuscle without the accompanying wave[4]? One might object, that we are here asking too artificial questions trying to carry to an extreme the "to the letter" understanding of the theory. However, *noblesse oblige*: if the theory is classical it should support the attempts of literal understanding.

One might still think about improving the completeness of the scheme by adding some more physical mechanisms. Thus, one could assume that, since the material point is governed by the Ψ field, then consistently, it may be captured only with the help of the Ψ field. This leads to the idea that the Schrödinger propagation equation for Ψ is only an approximate law, to be supplemented by some nonlinear wave mechanics which would make the field Ψ shrink from time to time: and only when the wave Ψ shrinks to a droplet, can the point particle inside be considered to be "captured." However, the completion of Schrödinger's wave mechanics in that direction would, in itself, be a completely new step of the theory, far more important than all previous hidden-parameter schemes. Moreover, it would make the whole aspect of Brownian motion irrelevant: because then the role of the "stochastic point" would be reduced to that of a dust particle circulating passively in the Ψ field and unnecessary even to explain the localization experiments. As a result, the part of the theory concerning the "Brownian motion" could be removed by Occam's razor and we would be left with a certain realistic interpretation of Ψ, with all its immanent

[4]Bohm (1952a, b, 1953) is aware of the dangers of the field interpretation and tries to neutralize them by assuming that each particle has its own Ψ field which is not perceived by the other particles of the same kind. This assumption, however, is not less artificial than the theory with "ghost waves."

difficulties which, once upon a time, have been raised against a field interpretation by Schrödinger (Jammer, 1966). We thus conclude that no consistent classical equivalent of the orthodox quantum mechanics has been offered in Nelson (1966). Paradoxally, we feel entitled to quote Nelson (1967) against Nelson (1966) himself: "Some theories, though mathematically correct, can be physically wrong." Concerning the stochastic theory in Nelson (1966), if not physically wrong, it is at least physically absent. This makes the following question even more intriguing: what precisely is the Nelson scheme? Is it only a formal art? Or, perhaps, an emerging fragment of a yet undiscovered theory?

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